Dynamics, Functional Equations, Infinite Combinatorics and Probability Conference 2017

In July 2012, we hosted a Mini-Conference on Infinite Combinatorics aimed at bringing together mathematicians researching in fields where a common underpinning was seen to be a theorem of Steinhaus.

Following the success of this event, the Department of Mathematics is pleased to announce a sequel conference with a wider remit:

Topological Dynamics, Functional Equations, Infinite Combinatorics and Probability Conference

Dates

12 – 14 June 2017, with registration from 10.00am and first sessions starting at 10.30am

Venues

- 12 – 13 June 2017 – CLM.4.02, Clement House, LSE (building "CLM" on the map)
- 14 June 2017 – CLM.3.02, Clement House, LSE (building "CLM" on the map)

Organisers

The principal organisers are Adam Ostaszewski (LSE) and Nick H. Bingham (Imperial & LSE).

An Advisory Panel includes Janusz Brzdęk (Krakow Pedagogical University) and Christopher Good (Birmingham) and the organisers.

Conference theme

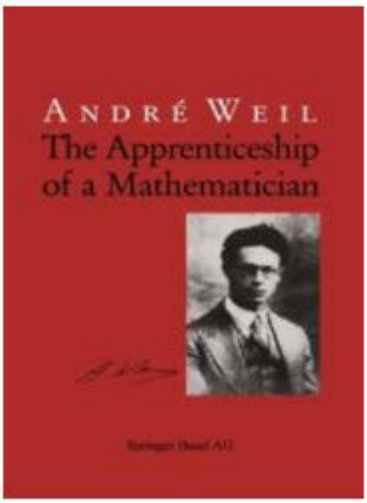
The classical Steinhaus-Weil property (SW) asserts that in a topological group a set S that is, under various possible definitions, non-negligible has the identity as an interior point of $S^{-1}S$. Through its many descendants SW is a foundational stone and, if not a unifying theme, then an inspirational one for several branches of mathematics: especially those in the title of the conference.

The conference's linking theme of Probability takes the wider `topological group and related structures' context (such as hypergroups – an established tool in the analysis of random walks): thereby stepping beyond the established topological vector spaces context. In the latter, SW emerges both in the regular variation of measures and in the subspaces of differentiability of measures (as in the Cameron-Martin spaces of Gaussian measures).

Earlier topological-group analysis of SW due to Solecki recognizes local forms of amenability and combinatorial features as either enabling SW or blocking it when negligibility is expressed via Christensen's Haar-nullity (i.e. in the absence of Haar measures). There are alternative combinatorial versions and density versions, some of them enunciated in the infinite combinatorics of the \beta N context as well as use of ample generics inspired by work in model theory. There is new work in all these areas to review and absorb.

Taken from the front cover of "Regular Variation" by Bingham, Goldie & Teugels (1989)

Hugo Steinhaus



Front cover of "The Apprenticeship of a Mathematician" by André Weil (1992)

Confirmed Participants

Peter Allen (LSE), **Dave Applebaum** (Sheffield), **Nick Bingham** (Imperial & LSE), **Vladimir Bogachev** (Moscow), **Janusz Brzdęk** (Kraków), **Peter Cameron** (St. Andrews & QMUL), **Charles Goldie** (Sussex), **Christopher Good** (Birmingham), **Eliza Jabłońska**(Rzeszów), **Imre Leader** (Cambridge), **Dugald MacPherson** (Leeds), **Adrian Mathias**(Réunion & UCL), **Harry Miller** (Sarajevo), **Adam Ostaszewski** (LSE), **Jaroslav Smítal** (Opava), **Marta Štefánková** (Opava), **Dona Strauss** (Leeds)

Programme

To view the full event programme, including schedule, please see below.

Attendance

Those interested are welcome to attend for all or any part of the event; it is hoped that many people will be able to attend all three days. Participation is free but **registration is compulsory on a FIRST REGISTERED, FIRST SERVED basis by email to Rebecca Lumb at R.C.Lumb@lse.ac.uk**.

Titles and Abstracts

Peter Allen (LSE) - *Limits of discrete structures*

What does it mean to say that a sequence of graphs, of growing sizes 'tends to a limit'? There are several different ways to answer this question; I will try to discuss a few of the answers and what they are good for, and move on to how one might ask and answer similar questions for other discrete structures.

This talk will be mainly a survey of other people's results, aiming at giving the big picture without too many technical details.

Dave Applebaum (Sheffield) - ***Some spectral properties of invariant Feller processes on compact symmetric spaces***

We study a class of Feller processes on a compact symmetric space G/K which are projections of Levy processes on G having K-bi-invariant laws. Here G is a compact Lie group and K is a closed subgroup of G. We show that the associated Feller semigroup has a discrete spectrum, where the eigenvalues are given by Gangolli's Levy-Khintchine formula, and the eigenvectors are spherical functions. We are then able to obtain a corresponding convergent series expansion for the transition density, when it exists and is continuous (and we have necessary and sufficient conditions for that). Examples of the class of processes we consider include Brownian motion on G/K, having the familiar heat kernel as its transition density, and also some processes obtained by subordination.

This is joint work with Trang Le Ngan.

Nick Bingham (Imperial & LSE) -***Regular variation: new variations on an old theme***

We survey the field of regular variation (RV), begun by Karamata in 1930, continued by (among others) Bojanic & Karamata (193), de Haan (1970 on), and summarised in the standard work:

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Since 2007, together with Adam Ostaszewski of the LSE, we have been working on a programme, `BinO' (`Bingham-Ostaszewski: 24 BinO papers, 10 O papers, 1 Bin paper, and a book to write). This has addressed the foundational question raised in BGT: measurability suffices; so does the Baire property; what is really needed?, and the contextual question: RV was originally real analysis, but other contexts are possible; what is the natural one?

It emerges that, contrary to the chronological order (measure, then (Baire) category), it is the category case that is primary. Recall the other classic book that we build on:
J. C. Oxtoby, Measure and category: A survey of the analogies between topological and measure spaces, Springer, 1971/1980.

Our book will be called “Category and Measure”.

Vladimir Bogachev (Moscow) - ***Analogs of the Cameron-Martin space for non-Gaussian measures***

The talk is concerned with various analogs of the classical object associated to a Gaussian measure: its Cameron-Martin space, in case of non-Gaussian measures on infinite-dimensional spaces.

Some recent results and open problems connected with this object for Gaussian and non-Gaussian measures will be discussed.

Janusz Brzdęk (Kraków) - *The generalized Steinhaus property*

Let (S,+) be a semigroup endowed with a topology and write
$A-B:=\{z\in S:(z+B)\cap A\neq\emptyset,\ A,B\subseteq S\}$.

Let *L* and *M* be two families of subsets of S. We present and discuss several applications in functional equations of the following generalized Steinhaus property: the interior of the set A-B is nonempty for $A \in L$ and $B \in M$.

Peter Cameron (St. Andrews & QMUL) - *Sum-free sets*

There is a natural bijection between the points of Cantor space and the set of sum-free subsets of the natural numbers, which allows the notions of measure and Baire category to be transferred to the latter set. These two notions give differing views of what a typical sum-free set looks like. For Baire category, the typical set has a universality property, with a consequence concerning automorphisms of Henson's universal triangle-free graph; it also has density zero. However, for measure, the density spectrum has a very interesting structure, and poses a number of unsolved questions. There are also interesting questions about rationality.

Christopher Good (Birmingham) - *Shifts of finite type as fundamental objects in the theory of shadowing*

Shifts of finite type and the notion of shadowing, or pseudo-orbit tracing, are powerful tools in the study of dynamical systems. In this talk we discuss a deep and fundamental relationship between these two concepts.

Let X be a compact totally disconnected space and $f:X\to X$ be a continuous map. We show that f has shadowing if and only if the system (f,X) is (conjugate to) the inverse limit of a directed system of shifts of finite type. In particular, this implies that, in the case that X is the Cantor set, f has shadowing if and only if (f,X) is the inverse limit of a sequence of shifts of finite type. Moreover, in the general compact metric case, where X is not necessarily totally disconnected, we prove that f has shadowing if and only if (f,X) is a factor of (i.e. semi-conjugate to) the inverse limit of a sequence of shifts of finite type by a quotient that `almost lifts pseudo-orbits' a natural property we define here.

This is joint work with Jonathan Meddaugh.

Eliza Jabłońska (Rzeszów) - *Properties of Haar meager sets, analogous to properties of Haar null sets*

In 2013 Darji [Dar] defined a σ-ideal of "small" sets in an abelian Polish group which is equivalent to the family of meager sets in a locally compact group. He was motivated by Christensen's paper [Chr] where the author defined Haar null set in an abelian Polish group in such a way that in a locally compact group it is equivalent to the notion of Haar measure zero set.

We present interesting properties of Haar meager sets, especially those which are analogous to properties of Haar null sets.

[Chr] J.P.R. Christensen, On sets of Haar measure zero in abelian Polish groups, Israel J. Math. 13 (1972), 255-260.
[Dar] U.B. Darji, On Haar meager sets, Topology Appl. 160 (2013), 2396-2400.

Imre Leader (Cambridge) - *Monochromatic infinite sumsets*

It is well-known that there is a finite colouring of the natural numbers such that no infinite set X has X + X (meaning the pairwise sums from X, allowing repetition) monochromatic. It is easy to extend this to the rationals. But what happens for larger spaces, like for example the reals?

We show (assuming CH) that there is such a finite colouring of the reals.

We also show that, if our space is `large enough', then for every finite colouring there does exist an infinite X with X + X monochromatic.

This is joint work with Neil Hindman and Dona Strauss, and also joint work with Paul Russell.

Dugald MacPherson (Leeds) - *Polish groups as automorphism groups*

The full symmetric group Sym(X) of a countably infinite set X carries a natural Polish group structure, with closed subgroups being exactly the automorphism groups of first order structures M with universe X. We say that such M is homogeneous (in the sense of Fraisse) if any isomorphism between finite substructures of M extends to an automorphism of M. I will discuss several issues concerning the Polish group structure on Aut(M) for homogeneous M: the work of Kechris-Pestov-Todorcevic and later others connecting topological dynamics to Ramsey theory; (non)-existence of `ample homogeneous generic automorphisms', and consequences; group-theoretic structure, e.g. simplicity.

Adrian Mathias (Réunion & UCL) - *Linking descriptive set theory to symbolic dynamics*

In my paper "Analytic sets under attack", MPCPS 138 (2005) pp 465-485, among other results concerning the iteration of the (backwards) shift function on Baire space, a recursive point was found for which the sequence of derived ω-limit sets does not stabilise until the first uncountable ordinal. This talk will present the main ideas in the hope that further development will follow.

Harry Miller (Sarajevo) - *Various gauges of the size of sets of reals*

In 1920 H. Steinhaus proved that if A has positive Lebesgue measure then A - A , the algebraic. difference set of A , contains an interval. There are many gauges of the size of sets of reals. The above theorem connects two different gauges of "bigness". Another gauge is the following. A is "large" if for each finite set F, A contains a subset F' similar to F. Other bigness concepts are: A is shift-compact , A is Cauchy equation large. Here we discuss relations between these various concepts.

(presentation to be delivered via Skype)

Adam Ostaszewski (LSE) - *Asymptotic group actions and their limits*

The theory of regular variation has two forms (BGT Ch. 1, 3): (K) due to Karamata, resting on the Cauchy functional equation, and (BKdH): Bojanić and Karamata, de Haan; Goldie (or Gołąb-Schinzel functional equation. We unify these via the `circle operation' of Popa and Javor, giving a unified proof to their hardest result, on quantifier weakening.

A less classical but very useful setting is Beurling slow and regular variation, originating in Beurling's approach to extending the Wiener Tauberian theory beyond convolutions. The resulting Wiener-Beurling Tauberian theorem handles the Borel, Riesz and Beurling summability methods.

This hinges on groups and group actions, and analysis, so limits. So a topological treatment is given.

Jaroslav Smítal (Opava) - ***Generalized Dhombres equation: interplay between functional equations and dynamical systems***

We consider the equation $f(xf(x))=\phi(f(x)),x>0$, where ϕ is given, and f is an unknown continuous function $(0,\infty)\to(0,\infty)$. This equation was studied in 1975 by J. Dhombres (with $\phi(y)=y^2$), later it was considered in many papers for other particular choices of ϕ, and since 2001 in about 10 papers for arbitrary continuous functions, from [1] through [3]. The main problem, classification of possible solutions and description of the structure of periodic points contained in the range of the solutions, was basically solved. This process involved methods from one-dimensional dynamics but also some new methods which could be useful in other problems, not only concerning the above equation. In the talk we provide a brief survey.

[1] P. Kahlig and J. Smítal, On a generalized Dhombres functional equation, Aequationes Math. 62 (2001), 18-29.
[2] J. Smítal, M. Štefánková, On regular solutions of the generalized Dhombres equation, Aequationes Math. 89 (2015), 57-61.
[3] L. Reich, J. Smítal, and M. Štefánková, On regular solutions of the generalized Dhombres equation II, Results Math. 67 (2015), 521-528.

Marta Štefánková (Opava) - ***On generic and dense chaos for maps induced on hyperspace***

A continuous map f on a compact metric space X induces in a natural way the map f on the hyperspace K(X) of all closed non-empty subsets of X. We study the question of transmission of chaos between f and f. We deal with generic, generic ε-, dense and dense ε-chaos for interval maps. We prove that all four types of chaos transmit from f to f, while the converse transmission from f to f is true for generic, generic ε- and dense ε-chaos. Moreover, the transmission of dense ε- and generic ε-chaos from f to f is true for maps on general compact metric spaces.

Dona Strauss (Leeds) - ***Subsets of*** **βN** ***which are not Borel***

Anyone who has worked with βN will not be surprised to learn that many subsets of this space are not topologically simple, even though they are very simple to define algebraically. I shall show that the following subsets of βN are not Borel: the set of idempotents, the smallest ideal, any principal proper right ideal, βN+βN.

This is joint work with Neil Hindman.

LSE THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

Dynamics, Functional Equations, Infinite Combinatorics & Probability Conference

12 - 14 June 2017

If attending multiple days, please retain this programme throughout

#LSEMaths @LSEMaths

Accessibility Map

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

- Disabled lift
- Lift
- Disabled parking (blue badge)
- Male accessible toilet
- Gender Neutral accessible toilets
- Disabled access
- Roads and Footpaths closed
- No entry
- Buildings under construction

Disabled access

After 6.30pm, please call Security Control on 020 7955 6200 to ensure that any disabled access doors are open.

Portable ramp for 20 Kingsway (**KSW** only) is located in entrance foyer. Please call 020 7955 6200 for Security staff to set up the ramp on request.

95A	**95 Aldwych** *Aldwych*
ALD	**Aldwych House** *Aldwych*
CLM	**Clement House** *Aldwych.*
COL	**Columbia House** *Aldwych*
CON	**Connaught House** *Aldwych*
COW	**Cowdray House** *Portugal Street*
KGS	**King's Chambers** *Portugal Street*
1KW	**1 Kingsway**
KSW	**20 Kingsway**
32L	**32 Lincoln's Inn Fields**
44L	**44 Lincoln's Inn Fields** (not occupied by LSE)
50L	**50 Lincoln's Inn Fields** *Portsmouth Street*
LCH	**Lincoln Chambers** *Portsmouth Street*
LAK	**Lakatos Building** *Portugal Street*
LRB	**Lionel Robbins Building, Library**
NAB	**New Academic Building** *Lincoln's Inn Fields*
OLD	**Old Building** *Houghton Street*
PAR	**Parish Hall** *Sheffield Street*
PEA	**Peacock Theatre** *Portugal Street*
POR	**1 Portsmouth Street**

QUE	**Queens House** *Lincoln's Inn Fields*
SAR	**Sardinia House** *Sardinia Street*
SAW	**Saw Swee Hock Student Centre** *Sheffield Street*
SHF	**Sheffield Street**

STC	**St Clement's** *Clare Market*
TW1	**Tower One** *Clement's Inn*
TW2	**Tower Two** *Clement's Inn*
TW3	**Tower Three** *Clement's Inn*

Student Services Centre
Ground floor, Old Building

Graham Wallas Room
OLD 5.25, Old Building

Hong Kong Theatre
Ground floor, Clement House

Old Theatre
Ground floor, Old Building

Shaw Library Sixth floor, Old Building

Sheikh Zayed Theatre
New Academic Building

Thai Theatre New Academic Building

The Wolfson Theatre
New Academic Building

The Venue
Basement, SAW

Vera Anstey Room
Between ground and first floor, Old Building
(Step free access is not available to this venue)

3 Tuns Ground floor, SAW

Bean Counter
Basement, 32 Lincoln's Inn Fields

Café 54
Ground floor, New Academic Building

Daily Grind Tower 1/2 Reception

Denning Learning Cafe
First floor, SAW

Fourth Floor Café Bar Old Building

Fourth Floor Restaurant
Old Building

George IV pub Portugal Street

LSE Garrick Ground floor, Columbia House

Mezzanine Café New Academic Building

Plaza Café John Watkins Plaza

Senior Common Room and Dining Room Fifth floor, Old Building

Student Common Room
Ground floor, King's Chambers

Weston Café Sixth floor, SAW

June 2015

CONTENTS

INFORMATION

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MONDAY 12 JUNE 2017

Schedule

The first day of the conference will be held in room CLM.4.02, Clement House, London School of Economics and Political Science, starting at 10.30am (please see campus map inside front cover for details). Everyone interested is welcome to attend any part of the event.

Time	Speaker	Presentation title
10:00	**Coffee and arrival**	
10.30 - 11.30	**Nick Bingham** (Imperial & LSE)	Regular variation: new variations on an old theme
11.45 - 12.45	**Jaroslav Smítal** (Opava)	Generalized Dhombres equation: interplay between functional equations and dynamical systems
12.45	**Lunch** *(own arrangements – options on campus and nearby)*	
14:00 - 15.00	**Vladimir Bogachev** (Moscow)	Analogs of the Cameron-Martin space for non-Gaussian measures
15:15 - 16.15	**Dave Applebaum** (Sheffield)	Some spectral properties of invariant Feller processes on compact symmetric spaces
16:15	**Afternoon refreshment break**	
16:45 - 17.45	**Christopher Good** (Birmingham)	Shifts of finite type as fundamental objects in the theory of shadowing
17:45	**Finish**	

TUESDAY 13 JUNE 2017

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10:00	**Coffee and arrival**	
10.30 - 11.30	**Peter Cameron** (St Andrews & QMUL)	Sum-free sets
11.45 - 12.45	**Imre Leader** (Cambridge)	Monochromatic infinite sumsets
12.45	**Lunch** *(own arrangements – options on campus and nearby)*	
14:00 - 15.00	**Dugald MacPherson** (Leeds)	Polish groups as automorphism groups
15:15 - 16.15	**Eliza Jabłońska** (Rzeszów)	Properties of Haar meager sets, analogous to properties of Haar null sets
16:15	**Afternoon refreshment break**	
16:45 - 17.45	**Janusz Brzdęk** (Kraków)	The generalized Steinhaus property
17:45	**Finish**	

Tuesday's conference will be followed by dinner at 6.15pm at **Thiwanya**, 10 Gate Street, Holborn, London, WC2A 3HP - www.thiwanya.co.uk. Please notify Adam Ostaszewski (A.J.Ostaszewski@lse.ac.uk) or Rebecca Lumb (r.c.lumb@lse.ac.uk) if you wish to attend.

WEDNESDAY 14 JUNE 2017

Schedule

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ABSTRACTS

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The full symmetric group Sym(X) of a countably infinite set X carries a natural Polish group structure, with closed subgroups being exactly the automorphism groups of first order structures M with universe X. We say that such M is homogeneous (in the sense of Fraisse) if any isomorphism between finite substructures of M extends to an automorphism of M. I will discuss several issues concerning the Polish group structure on Aut(M) for homogeneous M: the work of Kechris-Pestov-Todorcevic and later others connecting topological dynamics to Ramsey theory; (non)-existence of `ample homogeneous generic automorphisms', and consequences; group-theoretic structure, e.g. simplicity.

Adrian Mathias (Réunion & UCL) - *Linking descriptive set theory to symbolic dynamics*

In my paper "Analytic sets under attack", MPCPS 138 (2005) pp 465-485, among other results concerning the iteration of the (backwards) shift function on Baire space, a recursive point was found for which the sequence of derived ω-limit sets does not stabilise until the first uncountable ordinal. This talk will present the main ideas in the hope that further development will follow.

Harry Miller (Sarajevo) - *Various gauges of the size of sets of reals*

In 1920 H. Steinhaus proved that if A has positive Lebesgue measure then $A - A$, the algebraic difference set of A, contains an interval. There are many gauges of the size of sets of reals. The above theorem connects two different gauges of "bigness". Another gauge is the following. A is "large" if for each finite set F, A contains a subset F' similar to F. Other bigness concepts are: A is shift-compact, A is Cauchy equation large. Here we discuss relations between these various concepts.

(presentation to be delivered via Skype)

Adam Ostaszewski (LSE) - *Asymptotic group actions and their limits*

The theory of regular variation has two forms (BGT Ch. 1, 3): (K) due to Karamata, resting on the *Cauchy functional equation*, and (BKdH): Bojanić and Karamata, de Haan; *Goldie* (or *Gołąb-Schinzel functional equation*. We unify these via the `circle operation' of Popa and Javor, giving a unified proof to their hardest result, on *quantifier weakening*.

A less classical but very useful setting is *Beurling slow and regular variation*, originating in Beurling's approach to extending the Wiener Tauberian theory beyond convolutions. The resulting Wiener-Beurling Tauberian theorem handles the Borel, Riesz and Beurling summability methods.

This hinges on groups and group actions, and analysis, so limits. So a topological treatment is given.

Jaroslav Smítal (Opava) - *Generalized Dhombres equation: interplay between functional equations and dynamical systems*

We consider the equation $f(xf(x))=\varphi(f(x)),x>0$, where φ is given, and f is an unknown continuous function $(0,\infty)\to(0,\infty)$. This equation was studied in 1975 by J. Dhombres (with $\varphi(y)=y^2$), later it was considered in many papers for other particular choices of φ, and since 2001 in about 10 papers for arbitrary continuous functions, from [1] through [3]. The main problem, classification of possible solutions and description of the structure of periodic points contained in the range of the solutions, was basically solved. This process involved methods from one-dimensional dynamics but also some new methods which could be useful in other problems, not only concerning the above equation. In the talk we provide a brief survey.

[1] P. Kahlig and J. Smítal, On a generalized Dhombres functional equation, Aequationes Math. 62 (2001), 18-29.

[2] J. Smítal, M. Štefánková, On regular solutions of the generalized Dhombres equation, Aequationes Math. 89 (2015), 57-61.

[3] L. Reich, J. Smítal, and M. Štefánková, On regular solutions of the generalized Dhombres equation II, Results Math. 67 (2015), 521-528.

Marta Štefánková (Opava) - *On generic and dense chaos for maps induced on hyperspace*

A continuous map f on a compact metric space X induces in a natural way the map f on the hyperspace K(X) of all closed non-empty subsets of X. We study the question of transmission of chaos between f and f. We deal with generic, generic ε-, dense and dense ε-chaos for interval maps. We prove that all four types of chaos transmit from f to f, while the converse transmission from f to f is true for generic, generic ε- and dense ε-chaos. Moreover, the transmission of dense ε- and generic ε-chaos from f to f is true for maps on general compact metric spaces.

Dona Strauss (Leeds) - *Subsets of βN which are not Borel*

Anyone who has worked with βN will not be surprised to learn that many subsets of this space are not topologically simple, even though they are very simple to define algebraically. I shall show that the following subsets of βN are not Borel: the set of idempotents, the smallest ideal, any principal proper right ideal, βN+βN.

This is joint work with Neil Hindman.

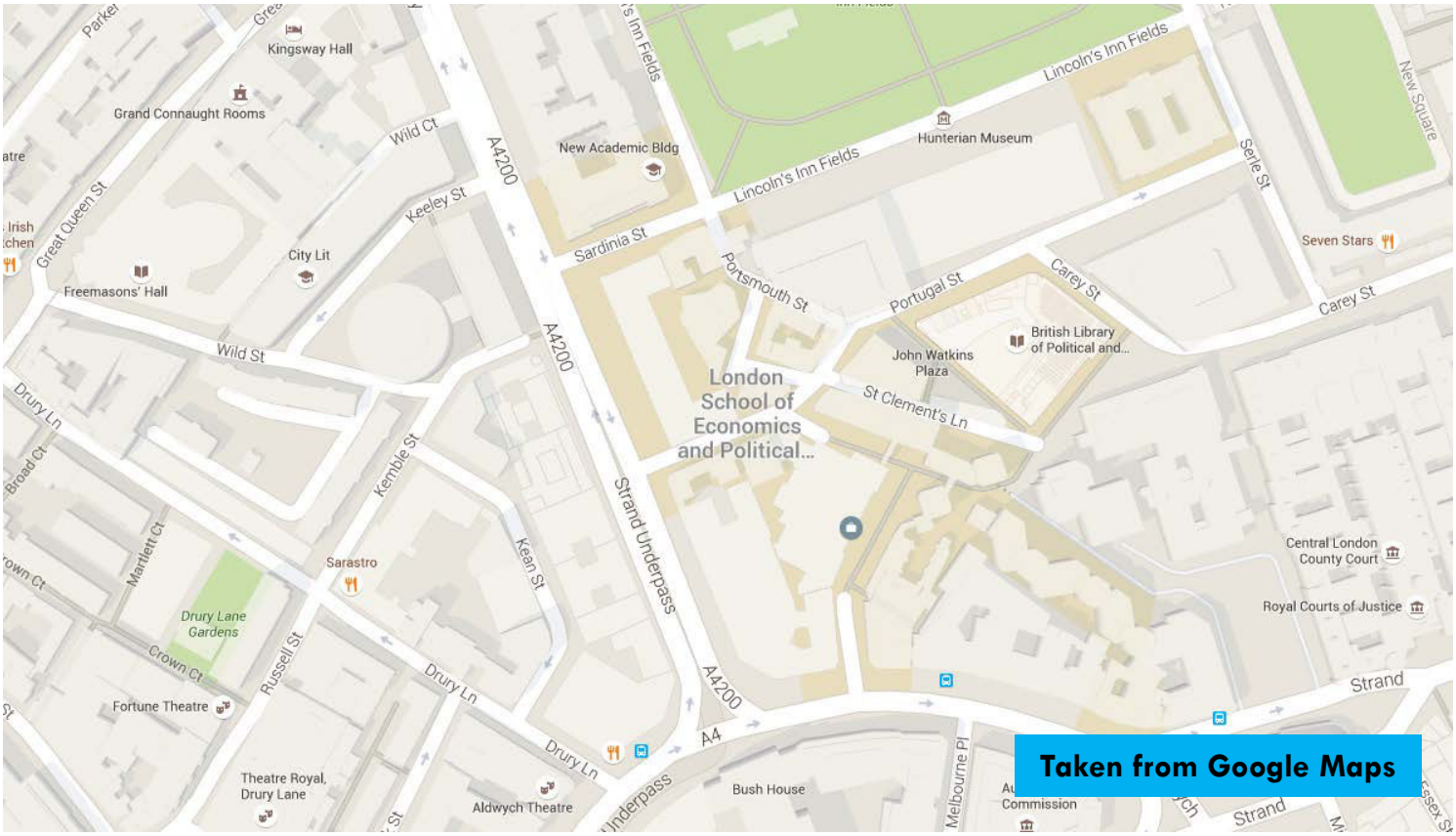
PLACES TO EAT: in and around LSE

Close by:

All Bar One – modern chain with full menu – 58 Kingsway
Belgo – Belgian beer & food – 67 Kingsway
Bill's – European chain with full menu – 42 Kingsway
Café Amici – Italian café – 7-9 Kingsway
Costa – standard café – 9-11 Kingsway
EAT – sandwich bar (chain) – 7-9 Kingsway
Paul – bakery & café – 36-38 Kingsway
Pret a Manger – standard café – 29 -33 Kingsway
Sainsburys – supermarket – 129-133 Kingsway
Shakespeare's Head – Wetherspoon's, standard pub food – 64-78 Kingsway
Starbucks – standard café – 10 Kingsway
Subway – fast food sandwich shop – 15 Kingsway
The Delaunay Counter – casual café-deli – 55 Aldwych
Viet Eat – Vietnamese – 48 Kingsway
Wasabi – Japanese chain serving bento boxes, sushi & hot food – 19 Kingsway

On campus:

The Bean Counter – café with hot & cold snacks – 32 Lincoln's Inn Fields
Café 54 – grab & go – New Academic Building
Fields Bar and Kitchen – perfect for a relaxed lunch – Lincoln's Inn Fields
Fourth Floor Café Bar – relaxing café with freshly made deli sandwiches – Old Building
Fourth Floor Restaurant – offers a wealth of eating options – Old Building
George IV Pub – perfect for a pub lunch – Portugal Street
LSE Garrick – cafe & restaurant – Columbia House
Mezzanine Café – pop up with different daily options – New Academic Building
Plaza Café – coffee and snacks – John Watkins Plaza



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